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EVALUATION OF LANDSAT-D ORBIT  
DETERMINATION USING A  
FILTER/SMOOTHER (PREFER)

BY  
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FINAL REPORT  
CONTRACT NAS 5-26807

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## 1.0 INTRODUCTION AND SUMMARY

This report summarizes the results of a study in which a filter/smoothen orbit determination program (PREFER) was used to refine the ephemerides produced by batch least squares orbit determination (GTDS). This is a follow-up to a previous study [1] in which an attempt was made to determine the optimal processing procedure for the LANDSAT-D orbit. However, in this study, more emphasis was placed on determining the robustness of the orbit determination in the presence of modeling errors.

Simulated range and range rate data for five tracking stations were first generated using GTDS. Then GTDS was used (in the differential correction mode) to produce a nominal trajectory which was input to PREFER. The GTDS differential correction (DC) run was made using models which differed from those used to produce the simulated data. These model differences were chosen to be fairly realistic approximations to the errors in the models actually used for operational orbit determination. Several different simulation runs were made with different types of model errors in order to determine the sensitivity to these errors.

The nominal trajectory and the simulated measurement data were input to PREFER to produce a smoothed ephemeris file. Numerous runs of PREFER were made in which parameters describing the statistics of the model errors were varied. The likelihood function computed by the Kalman filter was used to determine the "best" choice of input parameters. There was strong negative correlation between the likelihood function and the errors in the smoothed ephemeris. Thus, the use of the likelihood function to determine the optimum choice of input parameters was further validated.

## 2.0 TEST CASE

A simulated LANDSAT-D orbit was used as the test case in this study. The parameters of the LANDSAT orbit are:

Altitude - 700 km  
e - .0001  
inclination - 98.3°  
data period - 12 hours (7.3 revolutions)

The five USB tracking stations used in the study and their locations are:

#4 - Madrid, Spain  
#13 - Greenbelt, Maryland  
#30 - Orroral, Australia  
#31 - Fairbanks, Alaska  
#32 - Quito, Ecuador

## 2.1 Modeling Used in Data Simulation and Batch Orbit Determination

Table 2.1 lists the nominal differences between the models used in the data simulation and in the batch orbit determination. The model differences having the most effect upon the orbit determination are the measurement biases and different gravity models. The GEM9 (20,20) field is one of the best gravity models available while the WGS72 (16,16) is somewhat older and of lesser accuracy (although still quite good). The WGS72 model was chosen because it was derived from data completely independent of the data used in generating the GEM9 model. Thus, the differences between these two models should be an indication of the errors in the individual models (the difference between two models which were correlated would not be representative of the errors in the individual models).

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Table 2.1 Model Parameters of Test Case

Parameter	Model Description	
	Simulated Data	Batch Orbit Determination
data noise (range)	1 meter	1 meter (weighting)
data noise (range rate)	0.1 cm/sec	0.1 cm/sec (weighting)
gravitational field	GEM9 (20,20)	WGS72 (16,16)
measurement bias (R)	5 meters	---
measurement bias ( $\dot{R}$ )	0.1 cm/sec	---
station position errors	$\pm 5$ m each component	---
drag coefficient	2.0	1.5
density model	Flux table #150	Flux table #150
solar radiation pressure	yes	no
refraction-tropospheric	yes	yes
refraction-ionospheric	yes	yes
polar motion modeled	no	yes

The station position errors used in the data simulation were  $\pm 5$  meters for each component where the sign of the error was randomly assigned. The sign of the measurement biases was also randomly assigned. Other runs used biases of 15 meters and 0.5 cm/sec. Data noise of 5 meters and 0.5 cm/sec was also used in two runs.

Atmospheric drag at 700 km altitude and solar radiation pressure were almost negligible but the effects were included.

## 2.2 PREFER Statistical Models

The nominal input parameters used to describe the statistics of the modeling errors for PREFER are given in Table 2.2.

Table 2.2 Nominal Error Statistics Used in PREFER

Parameter	Value
<i>A priori</i> standard deviations	
$x, y, z$	20 meters
$\dot{x}, \dot{y}, \dot{z}$	2 cm/sec
gravitational accelerations	2.4(H), 0.7(C), 1.0(L) $\times 10^{-6}$ m/sec <sup>2</sup>
station measurement biases	5 meters (range)
	0.1 cm/sec (range rate)
station refraction	50 cm @ zenith
station position errors	5 meters (each component)
State Noise Spectral Density	
$x, y, z$	0
$\dot{x}, \dot{y}, \dot{z}$	0
Markov Process Standard Deviations	
gravitational accelerations	2.4(H), 0.7(C), 1.0(L) $\times 10^{-6}$ m/sec <sup>2</sup>
Time Constants of Markov Process	
gravitational accerleration	1200(H), 200(C), 200(L) seconds

It was originally intended that different refraction models were to be used in the simulation and batch orbit determination runs. However, there did not appear to be any method to force GTDS to use different models or model parameters: the refraction model used in the D.C. run exactly matched the model used in simulating the data and no error existed. Thus, the modeling error was introduced in PREFER using the following formula:

$$y'_m = y_m - 0.1 f_r \sin (2\pi t/86400)$$

where:  $y_m$  = the simulated value of the measurement (including all error sources)  
 $y'_m$  = the simulated value of the measurement with the additional refraction error included  
 $f_r$  = the refraction correction as listed on the Measurement Data File  
 $t$  = time of measurement relative to the epoch of the satellite ORBIT file (seconds)

Since the times of the passes of the satellite over the stations were different for each station, different refraction errors were used for each station. However, since only one-half day of tracking was used in our analysis, the sign of the refraction errors was always negative. In general, the magnitude of  $f_r$  was approximately 3 meters at zenith.

It was not realized until after many runs had been made that earth polar motion was not modeled in the GTDS simulation runs (the option to include polar motion had been requested on the input cards but it was not applied). Thus, there was an unintended model difference between the simulation and differential correction runs.

The differences between the "true" trajectory (used to generate the simulated data) and the "nominal" trajectory (obtained from the batch least squares fit) were somewhat larger in this study than in the previous study [1]. Thus, the differences between the PREFER results and



the "true" trajectory were also somewhat larger. It is believed that the polar motion modeling error in the GTDS runs is primarily responsible for these larger errors.

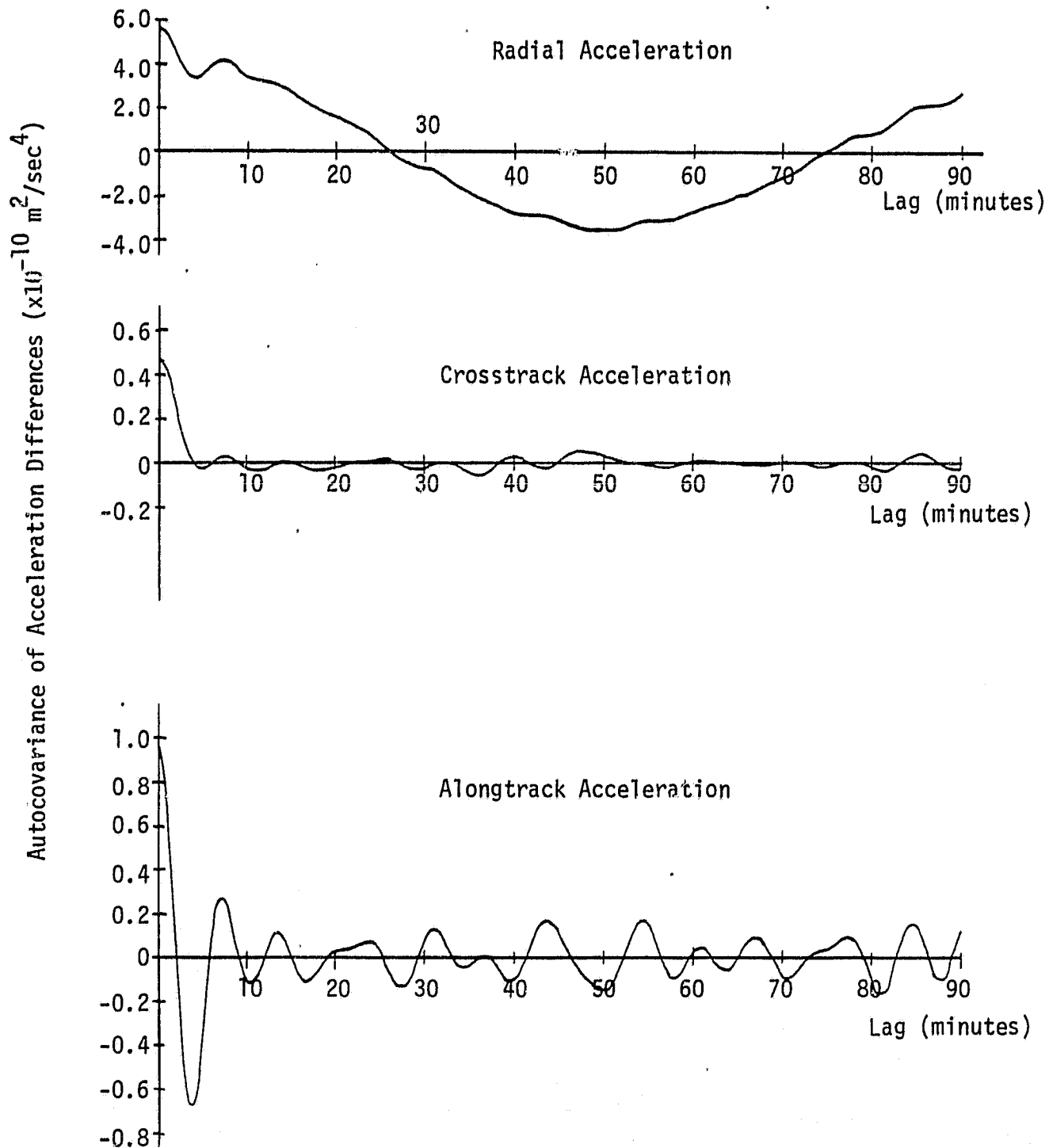
### 2.3 Covariance Function of Gravitational Acceleration

PREFER models the difference between the satellite acceleration as obtained from the nominal trajectory and the true acceleration due to gravitation as a first order Markov process. In the previous study [1], it was found that the steady state standard deviation of these perturbing accelerations was approximately  $3 \times 10^{-6}$  m/sec<sup>2</sup> when using time constants of 600 seconds (for each component). However, no attempt was made to determine the acceleration levels or the time constants separately for each of the three components (H,C,L).

We attempted, in this analysis, to estimate these acceleration levels and time constants for each component using the accelerations computed by GTDS. That is, two GTDS orbit generator runs were made and the trajectory information was stored on ORBIT files. One run used the gravity model (GEM9) and epoch orbital elements used in the data simulation. The other run used the WGS72 gravity model and epoch orbit elements obtained from the differential correction run. Thus, the first run generated the "true" trajectory while the second generated a trajectory close to the "nominal" (solar radiation and drag models were the same as for the "true" trajectory). At one minute intervals, the accelerations from each ORBIT file were transformed to local, HCL coordinates and the difference between the two files was computed for each component. Then a temporal autocovariance function was computed for the three components of this difference. Figure 2.1 displays the results for lags up to 90 minutes (approximately one orbital period).

All three components exhibit oscillatory behavior with a period of about seven minutes. This corresponds approximately to the truncation

Figure 2.1 Autocovariance Function of the Difference in Acceleration Between True Orbit and Nominal Orbit



level (degree and order 16) of the WGS72 field model. Presumably, a similar plot comparing the GEM9 (20,20) field model with the true gravitational field would show similar behavior but with a shorter period and smaller magnitude.

The radial component of the acceleration difference is considerably larger than the other components (as expected) and has a large oscillatory component with a period equal to the orbital period. It is interesting to note that as the epoch orbital elements of the second run (WGS72 model) are changed, the covariance function of the radial acceleration shifts up while the shape of curve remains constant. The other two components are not sensitive to the epoch orbital elements.

Although the first-order Markov acceleration models used in PREFER are not particularly good approximations to the autocovariance functions of Figure 2.1, we attempted to choose parameters of the first-order Markov process which would approximate these curves. The selected parameters are shown in Table 2.3.

Table 2.3 Selected Parameters of  
First Order Markov Acceleration Models

Component	Steady State Standard Deviation (m/sec <sup>2</sup> )	Time Constant (sec.)
radial	$24 \times 10^{-6}$	1200
crosstrack	$7 \times 10^{-6}$	200
alongtrack	$10 \times 10^{-6}$	200

Unfortunately, when we attempted to use the values of Table 2.3 in PREFER, we discovered that slightly better results (greater likelihood

function and smaller trajectory errors) were obtained when sigmas 5 to 10 times smaller than the values of Table 2.3 were used. Thus, the numbers in Table 2.2 were considered our nominal parameters. We can only guess that the smaller sigmas produced better results because the first order Markov process model is not a good approximation to Figure 2.1.

### 3.0 RESULTS

#### 3.1 Tabulation of Results

Table 3.1 lists the results of 18 runs which were made to investigate the sensitivity of the LANDSAT orbit determination to model errors. Of the runs on the "nominal" test case, run #4 has the highest likelihood function and its ephemeris errors are nearly the smallest. However run #2 is considered our "reference" run. A detailed description of the variations in the runs is given in the next section.

There are two different metrics which can be used to evaluate different Kalman filter solutions: the sum of weighted residuals and the log likelihood function. The log likelihood is computed as:

$$\ln(\text{likelihood}) = -1/2 \left[ \sum_{i=1}^n (z_i^T P_{z_i}^{-1} z_i + \ln |P_{z_i}|) \right] + \text{constant} \quad (3.1)$$

where:

$z_i$  is the  $i^{\text{th}}$  measurement residual vector  
 $P_{z_i}$  is the covariance matrix of the  $i^{\text{th}}$  measurement residual vector.

Since PREFER processes all measurements as uncorrelated scalars,  $z_i$  and  $P_{z_i}$  are scalars. These are computed normally by the filter so that the computation of the likelihood function is a trivial addition. The first summation in equation 3.1  $(\sum_{i=1}^n z_i^T P_{z_i}^{-1} z_i)$  is a sum of weighted residuals and should be Chi-squared distributed (mean of  $n$  with a standard deviation of  $\sqrt{2n}$ ) if all the error models are correct. If this quantity deviates substantially from  $n \pm \sqrt{2n}$ , then errors in the models, *a priori* statistics, measurement statistics, time constants and variances of Markov processes, etc. should be suspected.

Table 3.2 Summary of PREFER Results on LANDSAT-D Orbit

Run#	$\sigma_{\text{grav}}$ (m/sec <sup>2</sup> )	$\tau_{\text{grav}}$ (Seconds)	Edit Threshold	# Meas. Accepted	Sum of Weighted Residuals	Log Likelihood	Ephemeris Errors (meters)								Comments
							H		C		L		Total		
							RMS	Max.	RMS	Max.	RMS	Max.	RMS	Max.	
1	24., 7., 10. x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1019	10393	4.4	9.2	6.0	18.9	11.1	24.3	13.4	26.2	no refraction error
2	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1117	10406	3.8	8.1	7.0	13.1	11.4	24.9	13.9	26.0	
3	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1099	10415	3.5	7.4	7.0	13.0	10.7	22.3	13.3	23.7	
4	3.0, 3.0, 3.0 x 10 <sup>-6</sup>	600, 600, 600	5.5 $\sigma$	1082	1062	10413	3.9	8.7	6.5	21.3	11.6	29.0	13.9	31.0	
5	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1800, 1800, 1800	5.5 $\sigma$	1082	1108	10405	3.9	7.2	6.6	12.1	11.6	27.7	13.9	28.8	Refraction parameters not estimated
6	0.6, 0.2, 0.25 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1187	10387	4.6	13.6	9.5	17.4	14.3	47.0	17.8	47.2	
7	4.8, 1.4, 2.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1093	10406	3.7	7.3	6.0	12.8	11.2	25.8	13.2	27.3	
8	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1256	10368	5.1	12.4	6.4	12.8	14.8	44.9	15.9	44.9	
9	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	8.9 $\sigma$	1081	4951	8555	8.0	18.4	29.5	45.1	24.9	69.9	39.4	81.0	Meas. biases <u>not</u> estimated
10	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	961	1454	8978	8.0	19.9	4.5	9.6	24.8	69.0	26.5	69.2	station position error not estimated
11	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	8.9 $\sigma$	1082	1625	10171	7.9	22.5	4.3	9.7	24.5	78.5	26.1	78.6	Station position errors <u>not</u> estimated
12	12., 3.5, 5.0 x 10 <sup>-6</sup>	1200, 600, 600	6.3 $\sigma$	962	2789	8309	24.4	94.1	29.7	78.0	55.5	178.9	67.5	179.0	Nominal trajectory and and refraction have large errors
13	24., 7.0, 10. x 10 <sup>-6</sup>	1200, 600, 600	8.9 $\sigma$	1082	3461	9141	27.3	98.8	27.3	77.2	62.1	188.8	73.1	188.9	Nominal trajectory and refraction have large errors
14	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	6.3 $\sigma$	1082	1427	10251	5.5	11.7	5.3	11.4	17.8	47.1	19.4	47.3	Meas. biases = 15 m, 0.5 cm/sec
15	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	8.9 $\sigma$	994	17006	1527	5.2	10.3	10.5	18.1	15.8	40.7	19.6	40.7	Meas. noise = 5 m, 0.5 cm/sec
16	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1118	10412	3.8	8.1	7.2	13.2	11.4	25.0	14.0	26.2	priori $\sigma$ = 5 m, 0.5 cm/sec
17	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1116	10406	3.8	8.0	6.9	13.0	11.4	25.0	13.9	26.1	Mini-batch interval = 180 sec.
18	2.4, 0.7, 1.0 x 10 <sup>-6</sup>	1200, 200, 200	5.5 $\sigma$	1082	1117	10406	3.8	8.1	7.0	13.0	11.4	25.1	13.9	26.2	Mini-batch interval = 60 sec.

Note: Unless otherwise indicated, all runs estimate gravitational accelerations, measurement biases, refraction parameters and station position errors

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The log likelihood computed by the filter can be used to "fine tune" the models. The choice of model parameters which maximizes this function will usually yield the "best" filter/smoothen solution (i.e. smallest ephemeris errors). Notice that the maximization of the likelihood function involves minimizing the sum of weighted residuals and the residual covariance matrix. This differs from some batch orbit determination programs where only the sum of residuals is minimized.

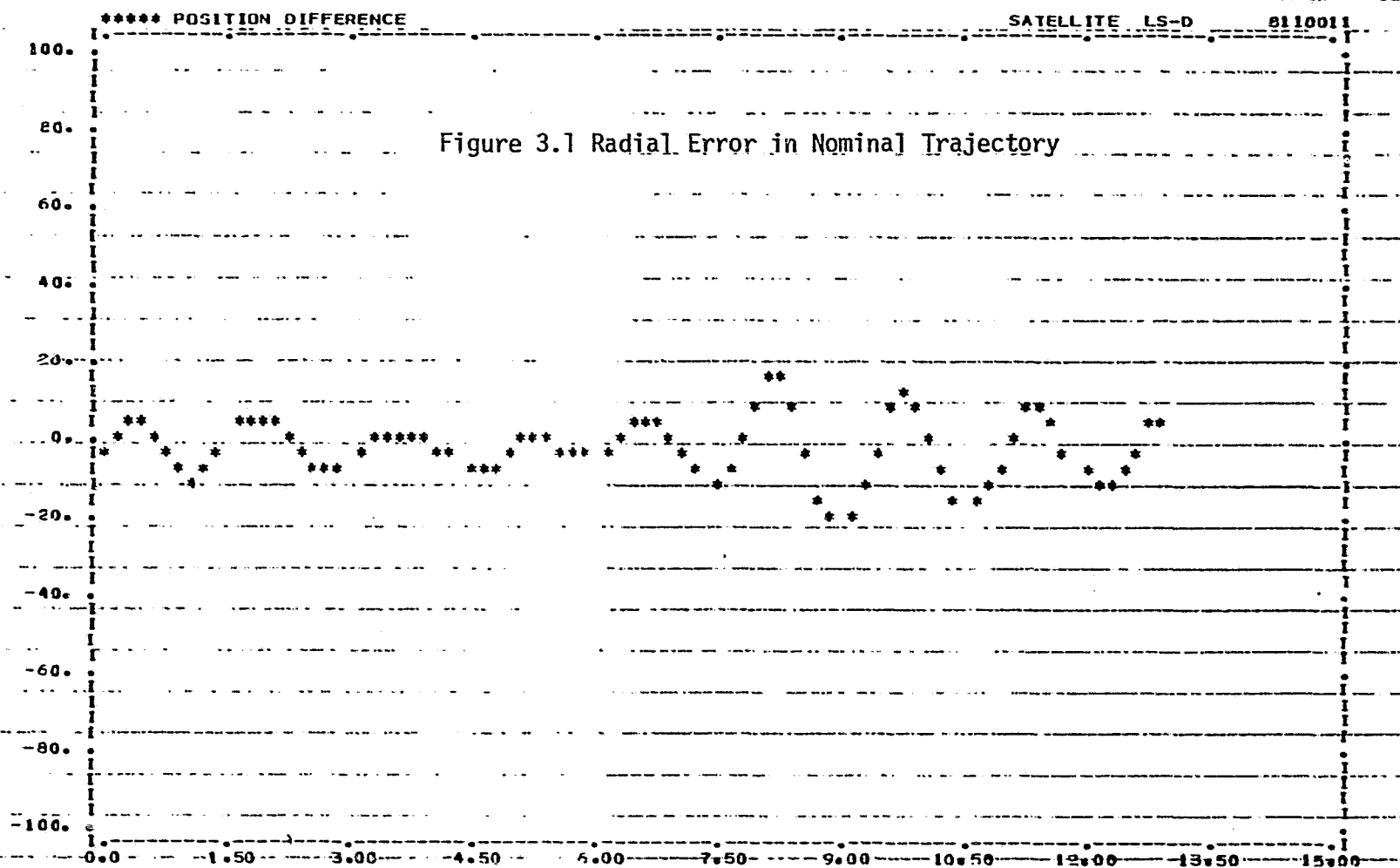
Other quantities listed in Table 3.1 are the errors in the smoothed ephemeris computed in radial (H), crosstrack (C), alongtrack (L) directions and the total (RSS) error. Both the RMS error along the trajectory and the maximum are listed.

Figures 3.1, 3.2 and 3.3 show the errors in the nominal trajectory. Notice that the ephemeris errors are relatively small in the first half of the trajectory and increase in the second half. As expected, the alongtrack error was the greatest. Figure 3.4 is a plot of the total (RSS) ephemeris error in the nominal trajectory and in the smoothed ephemeris for run #2 with the periods of ground tracking indicated. Notice that the ephemeris errors for both trajectories are largest during the data gaps after 470 minutes. Prior to 470 minutes, two or more ground stations were tracking the satellite but station 31 is the only one tracking after this time. It thus appears that one station is not sufficient to recover the orbit accurately. Notice, however, that the PREFER solution degrades much more slowly than the batch solution and it is consistently more accurate than the batch solution.

Figure 3.5 shows the three components of the PREFER ephemeris error and Figures 3.6 to 3.9 show the component of the analytically computed (from the smoother covariance matrix) standard deviations of the ephemeris error. In general the agreement between the actual error and the standard deviations is good, thus giving us confidence that the errors sources are modeled properly.

RADIAL DIFFERENCES IN METERS

Figure 3.1 Radial Error in Nominal Trajectory

TIME FROM YYMMDD HHMMSS IN HRS  
811001 195500

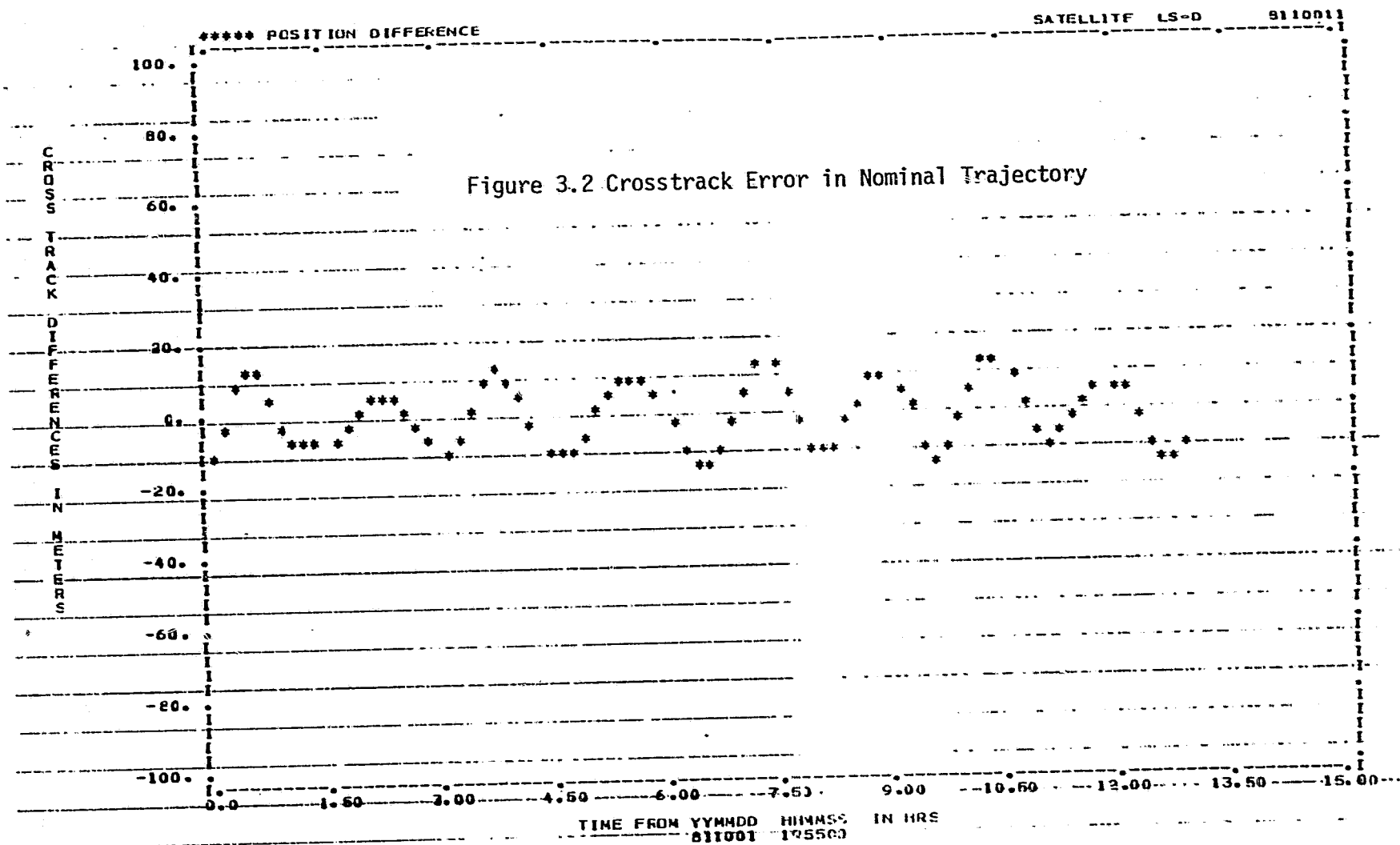
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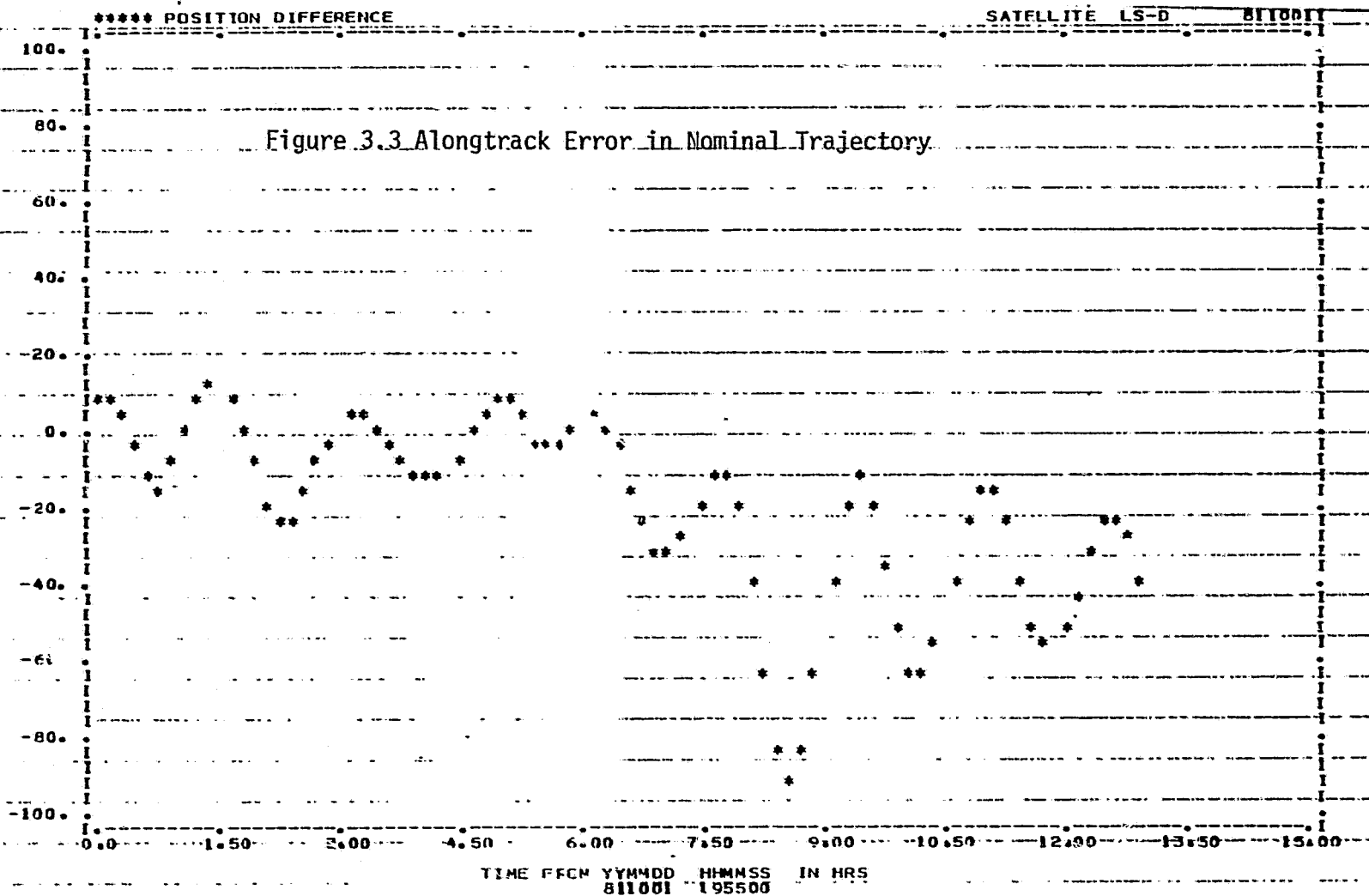


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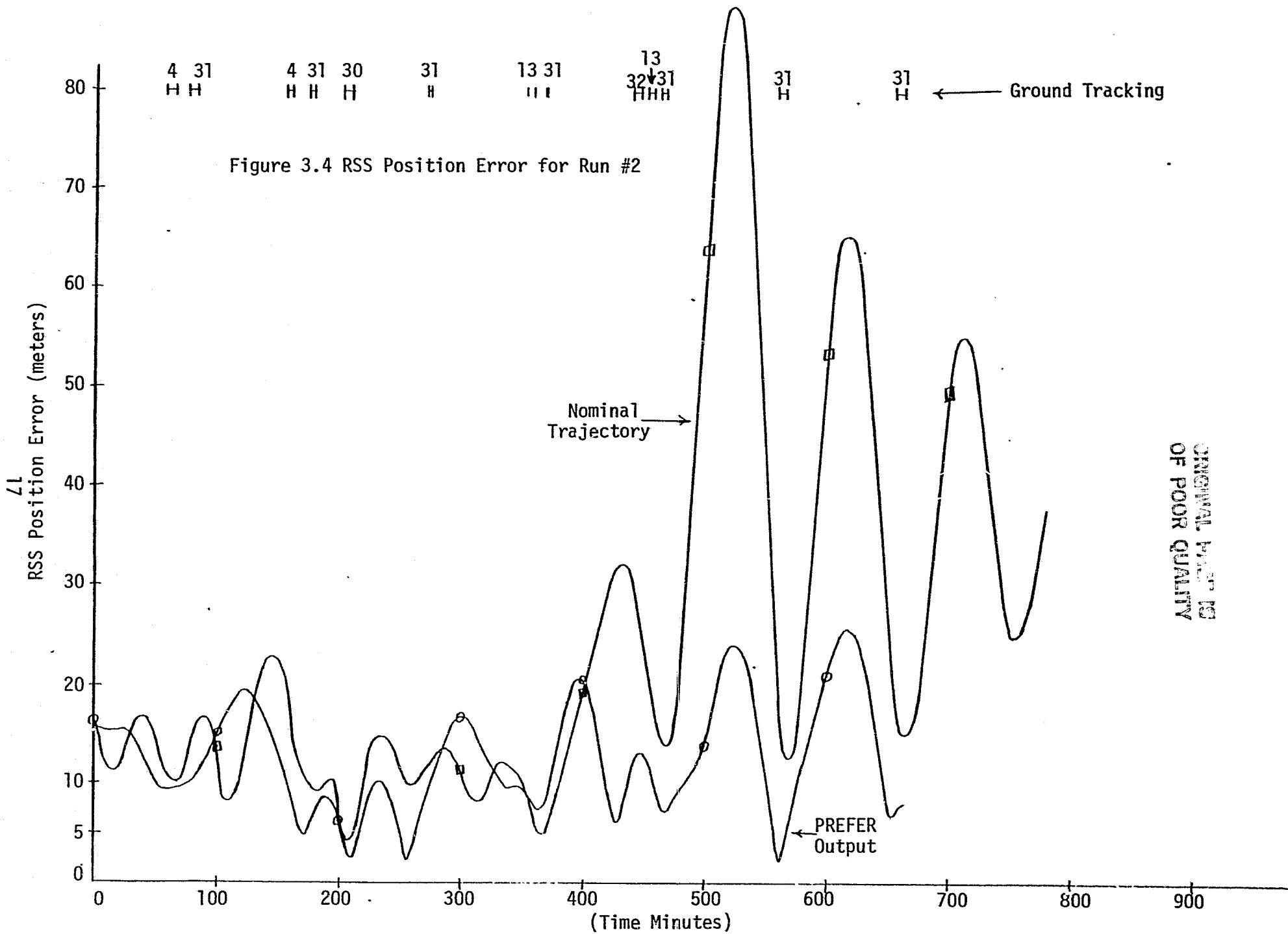
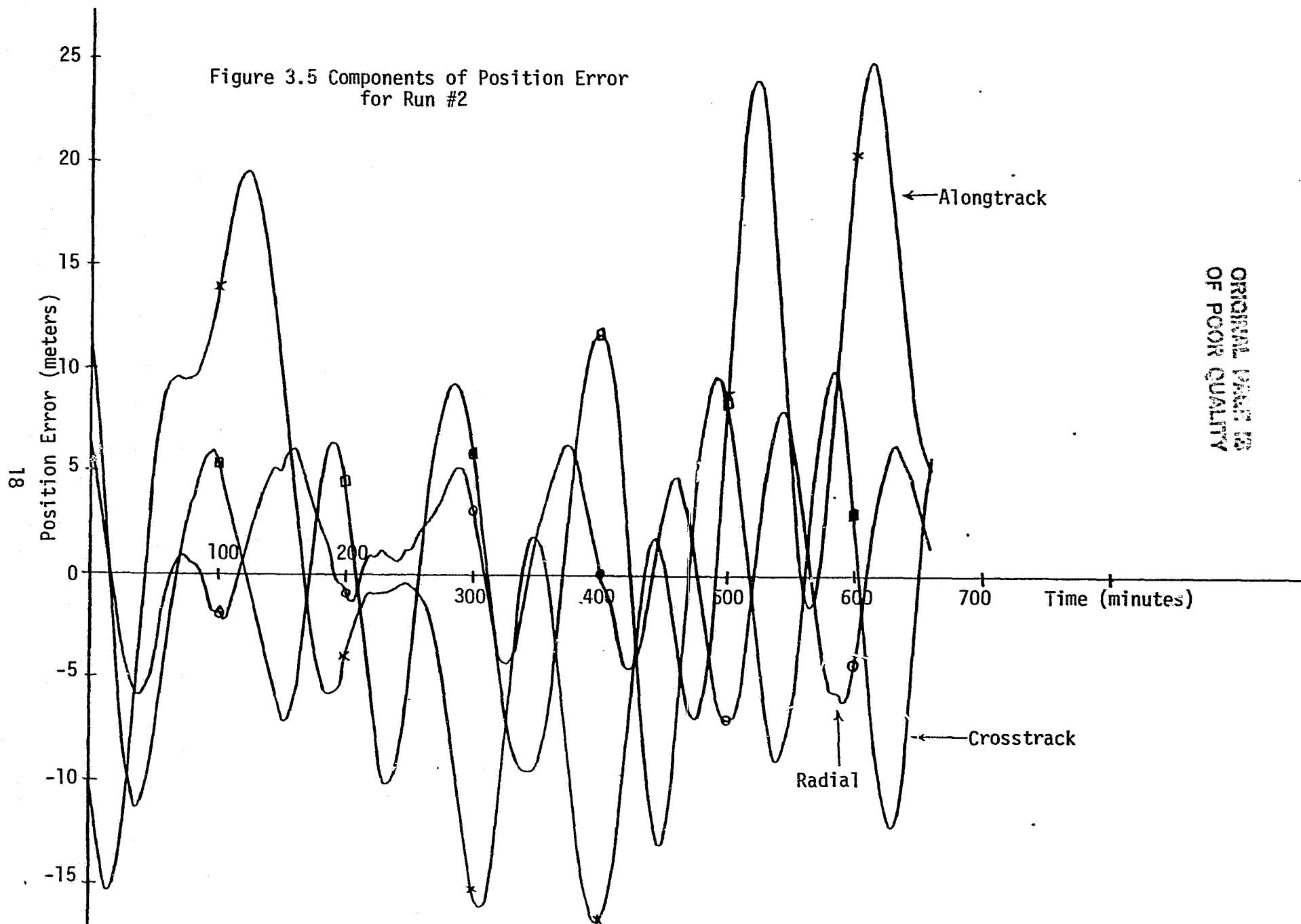
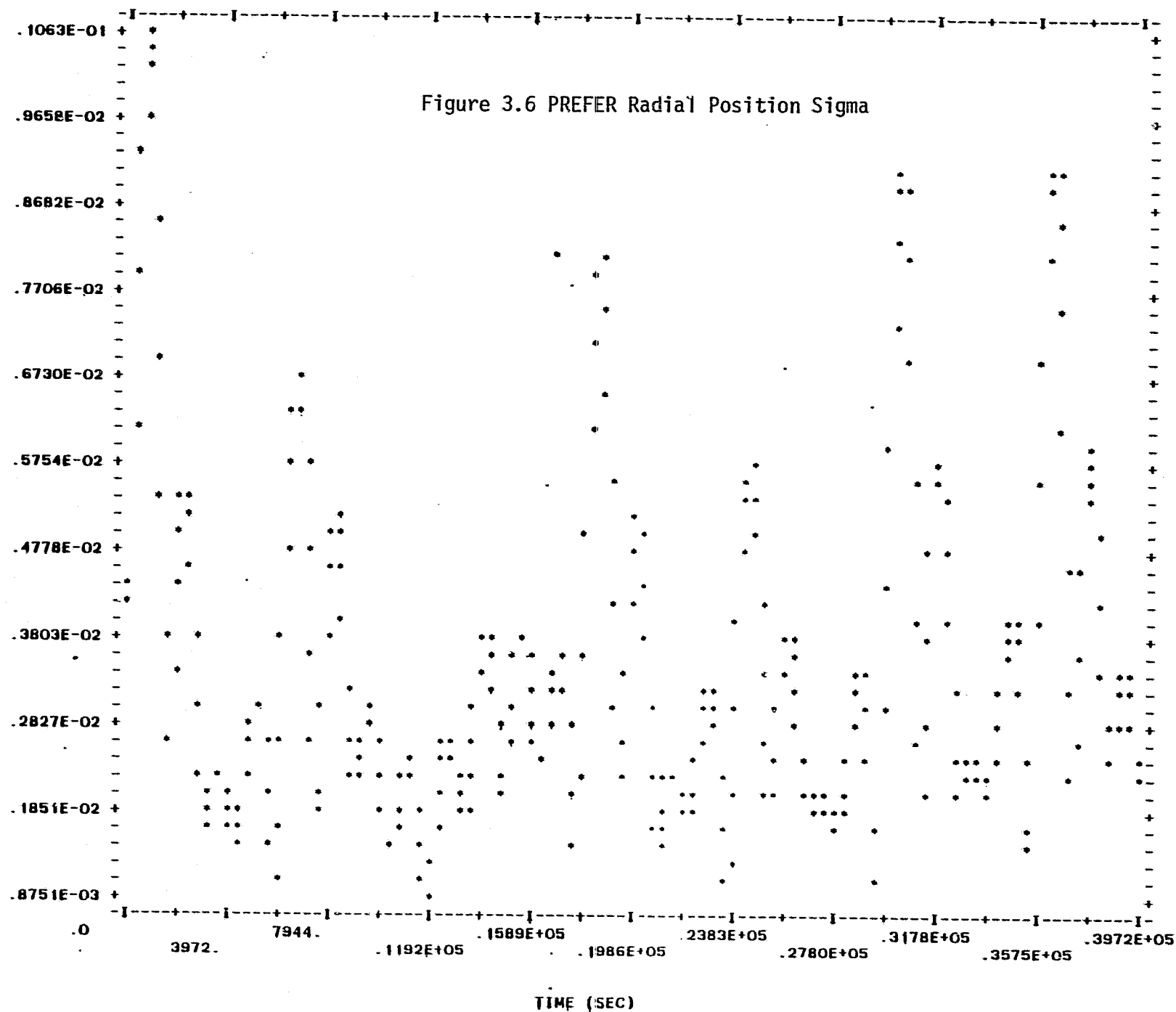


Figure 3.5 Components of Position Error  
for Run #2

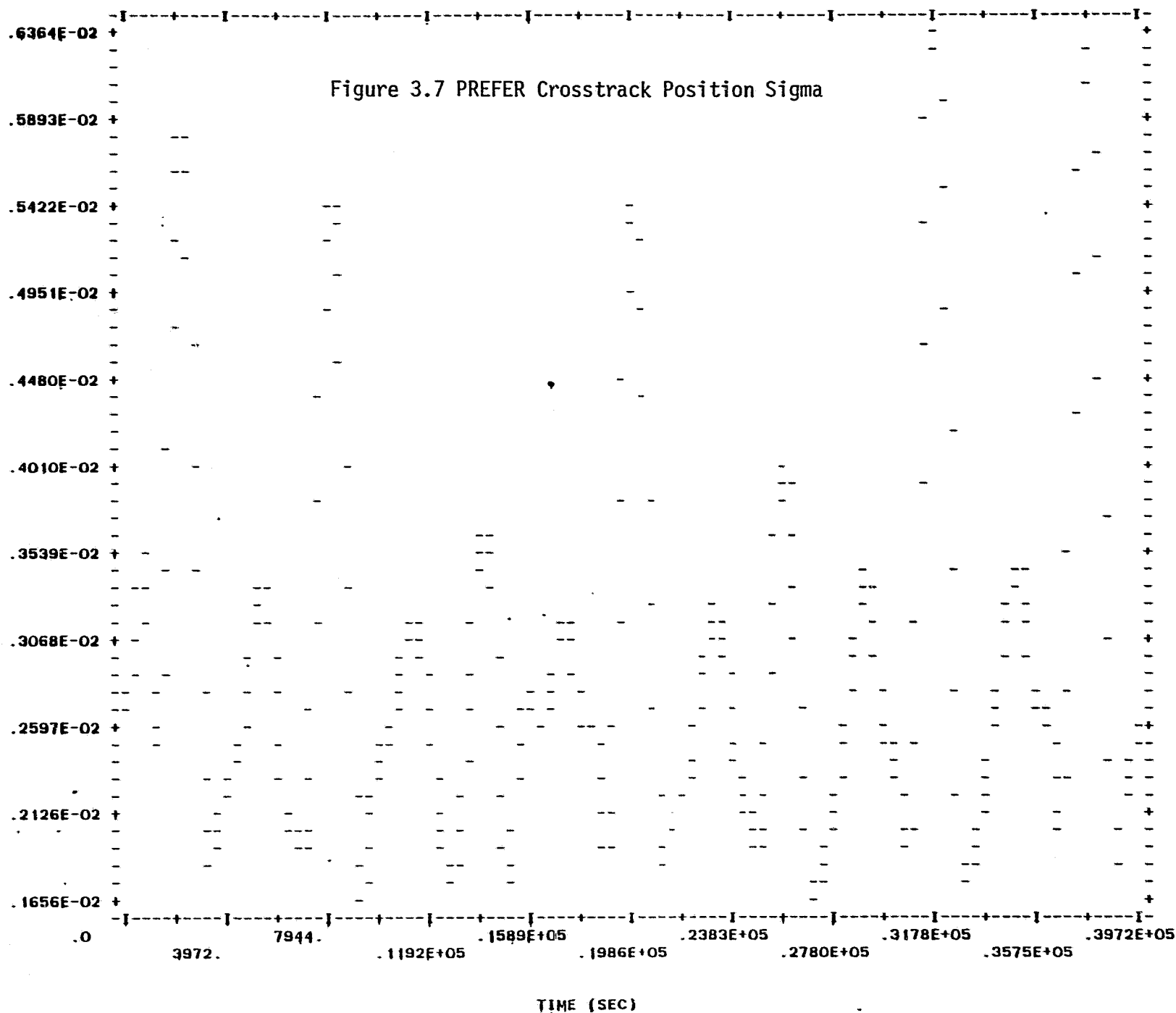




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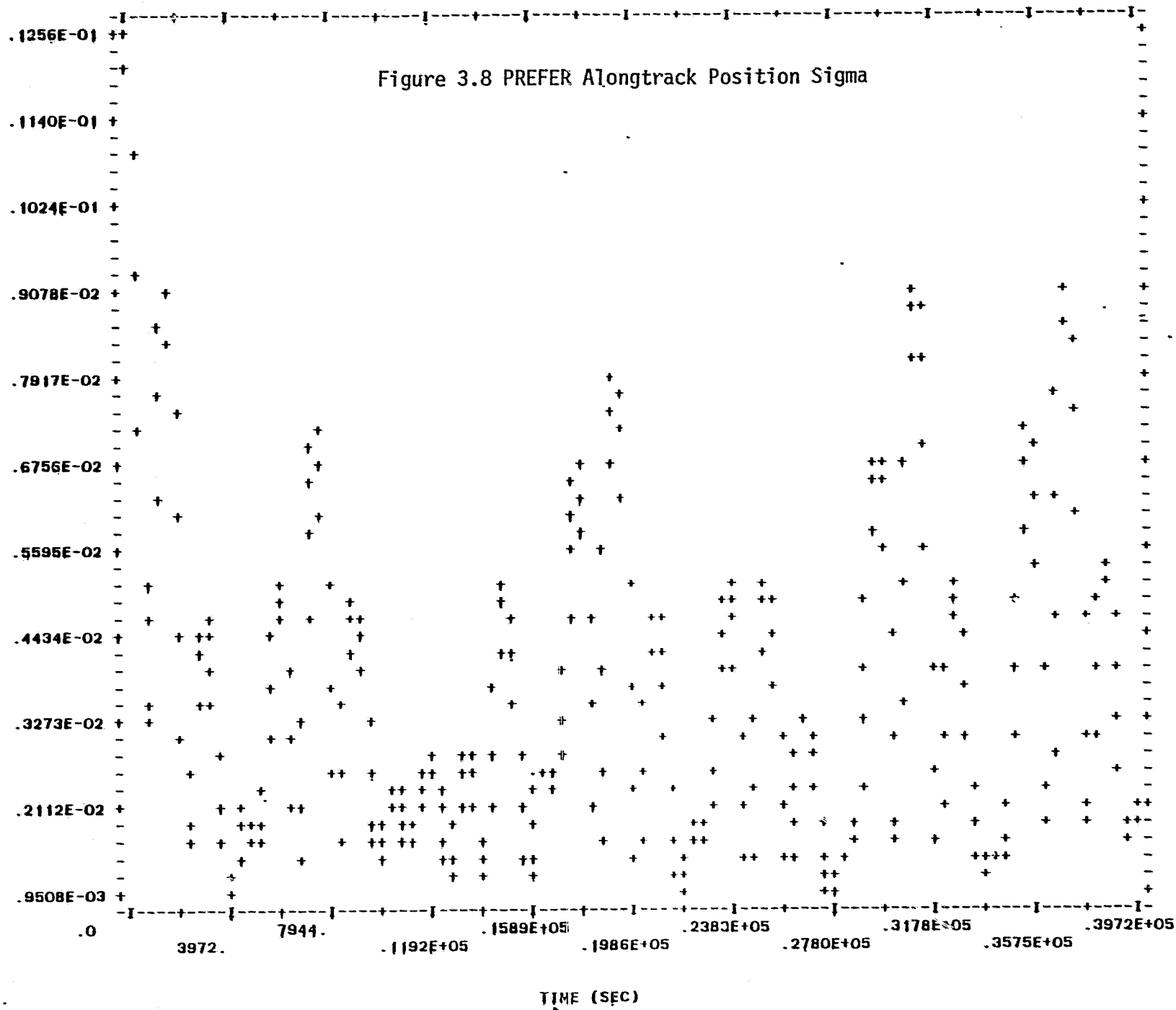
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Figure 3.7 PREFER Crosstrack Position Sigma

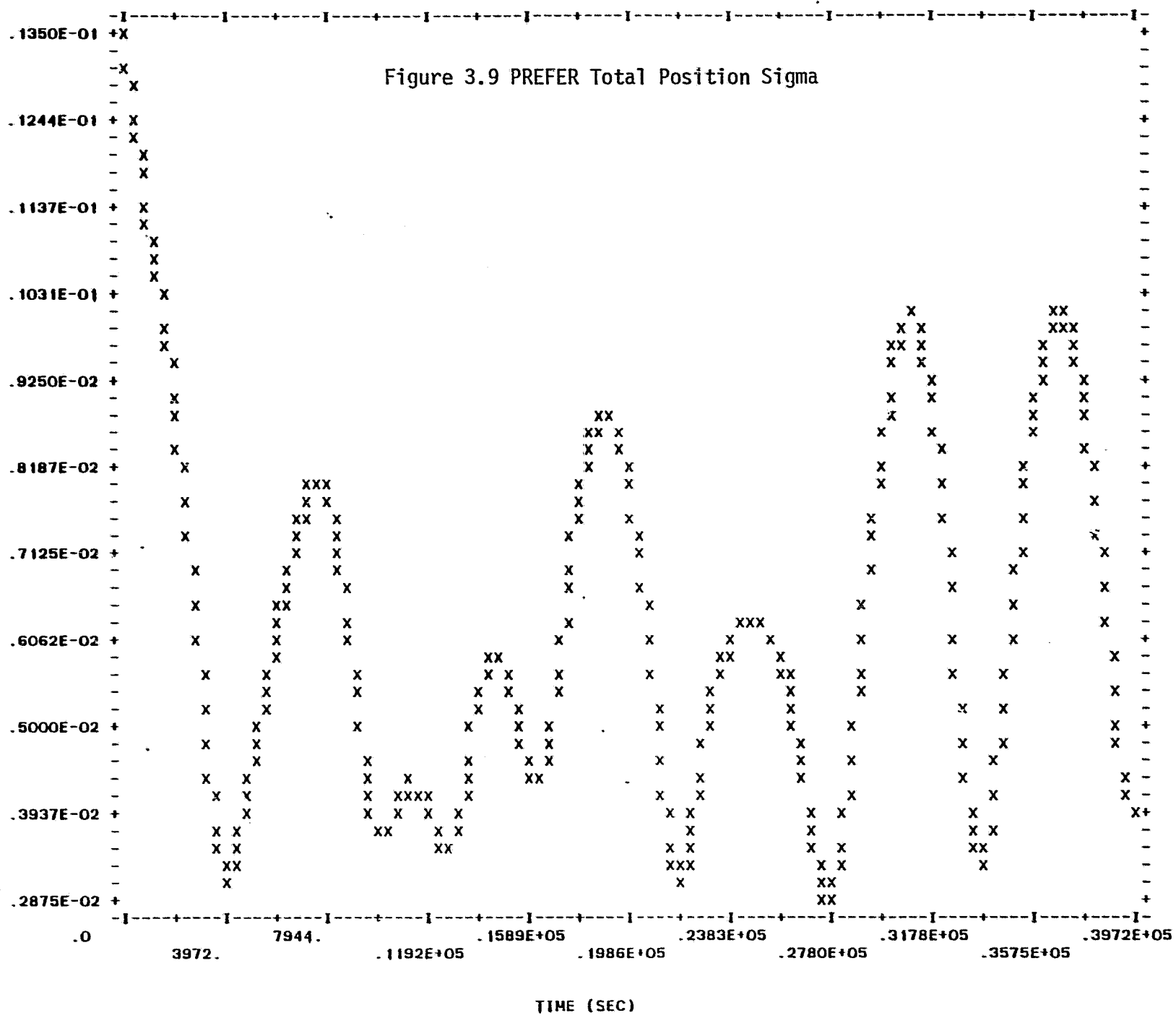
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Figure 3.8 PREFER Alongtrack Position Sigma

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TIME (SEC)



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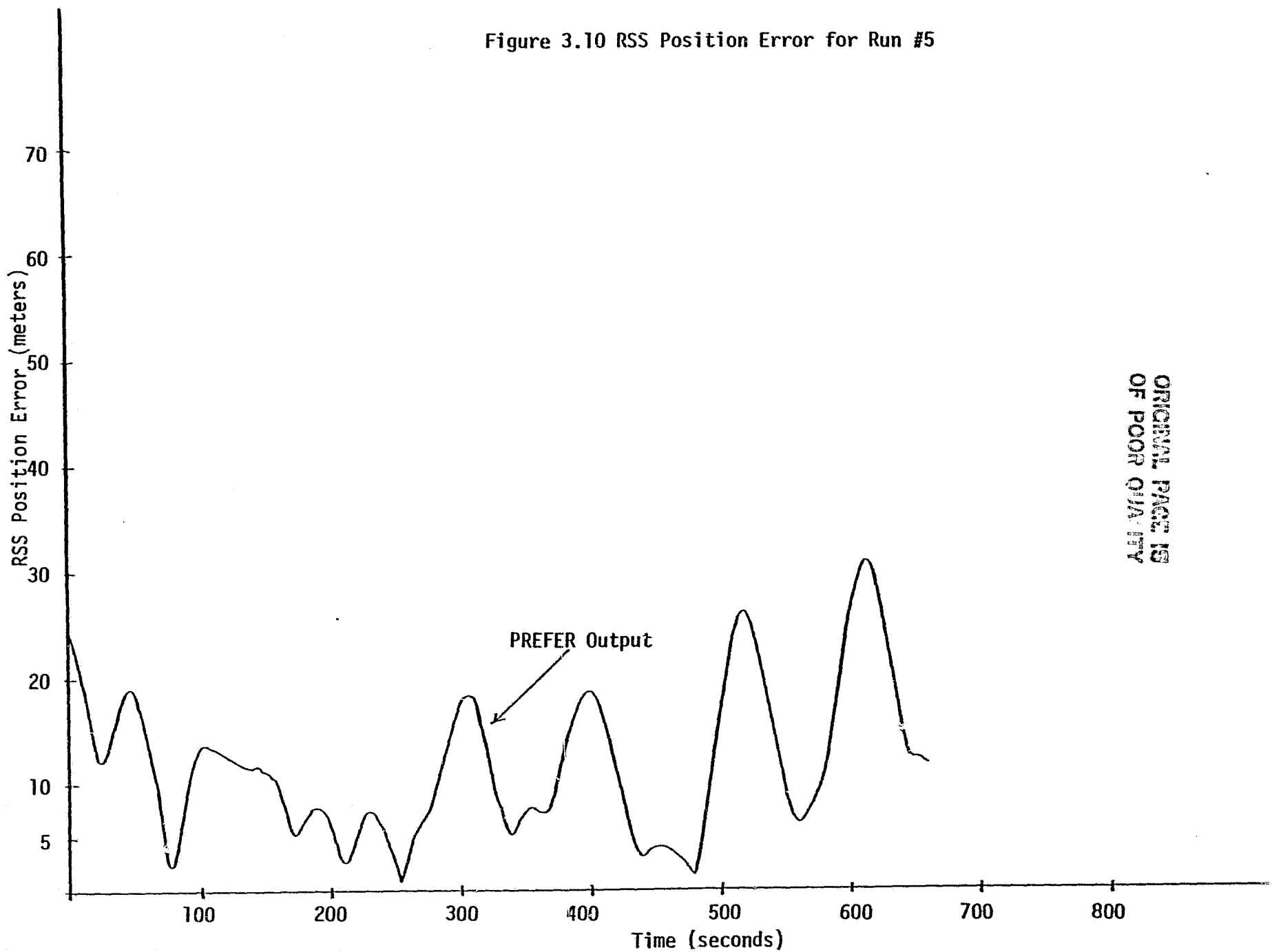
### 3.2 Sensitivity of Results to Error Models

#### 3.2.1 Gravitational Acceleration

The sensitivity of the PREFER solution to variations in the Markov models of gravitational acceleration are examined in runs 1, 2, 4, 5, 6 and 7. As mentioned before, run #2 is treated as our reference run. Run #1 uses the large, steady state standard deviations of Table 2.3 (In all of these runs, the *a priori* sigma was set equal to the steady state sigma of the Markov process.) Compared to run #2, run #1 has a smaller likelihood function and slightly larger ephemeris errors. Run #7 uses values between runs #1 and #2 but the results are close to those of run #2 (the likelihood is identical). In run #6, the sigmas were one-fourth those of #2 and the results are definitely much worse. Run #4 uses the sigmas and time constants found to produce the best results in the previous study ( $3 \times 10^{-6}$  m/sec<sup>2</sup> and 600 seconds for all components). The likelihood function of this run is greater than that of run #2 but the ephemeris errors are slightly larger. Finally, run #5 uses the sigmas of run #2 but time constants of 1800 seconds for all components. The results are again similar to run #2.

From these six runs, we conclude that the PREFER results are not sensitive to minor variations in the gravitational acceleration model provided that the model is a reasonably close approximation to truth. This is simply a statement of the fact that gradients are near zero in the neighborhood of an optimal solution. However, we have noted that the characteristics of the resulting ephemeris errors do change as the acceleration model is varied. For example, Figure 3.10 is a plot of the PREFER ephemeris error for run #5 (using  $3 \times 10^{-6}$  m/sec<sup>2</sup> and 600 seconds). Note that compared to run #2, (Figure 3.4), the ephemeris errors are smaller for longer periods of time but the peak errors are larger. Thus it is difficult to decide which is the "better" solution.

Figure 3.10 RSS Position Error for Run #5



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### 3.2.2 Station Position Errors

Runs 10 and 11 did not adjust station position errors (now modeled in earth-fixed coordinates). Run 10 used a measurement editing threshold of  $5.5\sigma$  with the result that 121 measurements were edited. Thus the editing threshold was increased to  $8.9\sigma$  in run 11 and all measurements were accepted. Compared to run #2, the log likelihood function of run 11 is significantly lower ( $\Delta=235$ ) and the ephemeris errors are considerably worse. Surprisingly, the ephemeris errors are slightly smaller in run 10 than in run 11.

Even though the modeling of station position errors has been changed from earth-centered inertial coordinates (in the previous study) to earth-centered fixed coordinates, most of the comments in Reference [1] concerning station position errors are still valid:

- a) The adjustment of station position errors is important
- b) The smoother estimates of station position errors are not consistent from one pass to the next (see Table 3.2)
- c) The smoother sigmas for the station position errors were not significantly reduced from the *a priori* (e.g. 4 meters versus 5 meters *a priori*)
- d) It is suspected that the benefit derived from adjusting the station positions arises from the decreased "gain" of the filter.

### 3.2.3 Measurement Biases

In run 9, measurement biases were not estimated. The decrease in the log likelihood was 1851 compared to run #2 and the ephemeris errors were extremely large. Notice, also that it was necessary to increase the editing threshold to 8.9 sigma.

Table 3.2 PREFER Estimate of Pass Parameters for Run #2

Time of Pass (sec.)	Station #	Meas. Bias		Refraction @ Zenith (m)	Station Position Errors (m)		
		R (m)	$\dot{R}$ (cm/sec)		x	y	z
3360	4	5.36	.092	.012	-4.69	4.75	3.07
4560	31	-4.96	.055	-.011	-12.33	16.97	-12.26
9360	4	8.11	.072	.00	2.01	-10.39	-.57
10440	31	-4.75	.182	-.112	-.28	3.24	-2.72
12240	30	4.14	-.19	-.131	-.39	2.91	-4.88
16320	31	-5.17	.064	-.226	.80	2.49	1.60
21240	13	5.05	-.057	-.151	-2.18	-3.08	-5.05
22080	31	-2.24	.015	-.294	-1.78	-1.96	-1.03
26400	32	-5.70	.105	-.233	-1.55	-8.48	6.00
27120	13	5.95	-.100	-.141	-8.02	-4.47	-6.87
27840	31	-4.37	.147	-.089	-3.27	-1.54	-7.76
33480	31	-5.06	.058	-.130	-7.41	.09	-3.44
39360	31	-5.24	.052	-.057	2.22	-.91	-3.39

In run #14, the simulated measurement data contained biases of 15 meters (range) and 0.5 cm/sec (range rate). This same data was also used in the GTDS differential correction run to obtain the nominal trajectory used by PREFER. However, PREFER used the nominal *a priori* standard deviations (5 meters, 0.1 cm/sec.) for the biases so that the actual errors were 3 to 5 times larger than the assumed errors. The results for run #14 were much worse than those for run #2 but not nearly as bad as those of run #9. The log likelihood of run #14 was also lower ( $\Delta=155$ ) than that of run #2.

Thus, we conclude that the adjustment of measurement biases is important and the *a priori* sigma used in PREFER should be a reasonable approximation to the actual biases. It is probably better to overestimate the magnitude of the biases rather than to underestimate them. The measurement biases appear to be quite observable since the recovered values in run #2 were usually within 1 meter and .05 cm/sec of the true values (Table 3.2)

#### 3.2.4 Refraction Parameters

The refraction parameters were not estimated in run #8. Compared to run #2, the log likelihood was 38 lower and the alongtrack ephemeris errors were much larger. The radial error also increased somewhat. In order to better evaluate the effects of refraction errors on the PREFER solution, run #3 did not include any refraction errors. There was little difference in the resulting smoother ephemeris errors but the log likelihood increased by 9 compared to run #2. Thus, we conclude that the estimation of refraction parameters is important and that no problems will arise if these parameters are estimated when modeling errors are not actually present.

The smoother estimates of the refraction parameter obtained from run #2 were less than 0.29 cm in magnitude at zenith (Table 3.2). Although we do not know the actual magnitude of the refraction corrections (they are not printed in the GTDS simulation runs), it is believed that the

maximum refraction error as used PREFER is about 0.30 meters at zenith. Thus, the estimates listed in Table 3.2 would appear to differ from the true values by no more than 0.15 meters (recall that the error varies sinusoidally with time). However, the smoother sigmas varied from 0.03 to 0.20 meters so the estimation errors may represent three sigma errors.

### 3.2.5 Measurement Noise

In run #15, the measurement noise of the GTDS simulated data was increased to 5 meters (range) and 0.5 cm/second (range rate). Since PREFER used *a priori* standard deviations of 1 meter and 0.1 cm/second, these errors represent 5 sigma modeling errors. Notice that considerable measurement editing occurred (even though the editing threshold was increased), the sum of weighted residuals increased an order of magnitude and the log likelihood decreased an order of magnitude. Although the smoother ephemeris errors were much worse than those of run #2, they were not as bad as might be expected from the log likelihood. Thus, we conclude that a slight mismatch between the true noise level and the assumed noise sigma will not have a large impact upon the results.

### 3.2.6 Large Errors in the Nominal Trajectory

It is generally expected that the GTDS differential correction run (which generates the nominal trajectory for PREFER) will use models as realistic as possible so that the errors in the nominal trajectory are no greater than 50 to 100 meters. In order to determine the degradation in the PREFER solution due to larger errors in the nominal trajectory, a nominal trajectory was created (via GTDS) using the WGS72 field model truncated at degree and order 4. Furthermore, the GTDS DC run did not correct for refraction so that net error in the refraction model was approximately 3 meters at zenith. The resulting nominal trajectory had RMS/maximum errors of 59/141 meters (radial), 34/87 meters (crosstrack) and 213/539 meters (alongtrack). It should be noted that the modeling errors in this GTDS run were quite extreme. Thus, the ephemeris errors are much larger than would be expected using any reasonable models.

Runs #12 and #13 (Table 3.1) used this anomalous nominal trajectory. In both of these runs, the sigmas of the gravitational accelerations and of the refraction parameters were increased: the standard deviation of the refraction parameter was 3 meters at zenith. No measurement editing occurred in run #13 but approximately 10% of the data was edited in run #12 because of the smaller editing threshold and smaller sigmas on the gravitational accelerations. Somewhat surprisingly, the results of run #12 were slightly better than those of run #13 but both were disappointing: the maximum alongtrack error was 189 meters. Although the PREFER solution is a great improvement over the nominal trajectory, the ephemeris errors are still quite large. It would appear that more effort is required to find a more suitable model for these large gravitational field model errors. This emphasizes the importance of using realistic models in GTDS so that the errors in the nominal trajectory are not excessively large.

#### 3.2.7 A Priori Statistics

Run #16 used *a priori* standard deviations on the orbital elements which were approximately one-third as large as the actual errors in the *a priori* orbital elements. Thus, PREFER is weighting the *a priori* orbital elements much more than it should be (compared to the measurement data). Fortunately the results are not sensitive to this form of modeling error: the ephemeris errors are almost identical to those of run #2. The log likelihood of run #16 is slightly larger than that of run #2 but we believe that this is somewhat of an anomaly.

#### 3.2.8 Mini-Batch Interval

In runs #17 and #18, the step size of the mini-batch interval was varied from the nominal 120 seconds to 60 and 180 seconds. Again the results were almost identical to those of run #2. Since the Markov time constants of the crosstrack and alongtrack accelerations are only 200 seconds, the assumption of deterministic dynamics within the mini-batches begins to break down when using mini-batch intervals of 180 seconds. Fortunately, the PREFER solution does not appear to be sensitive to this.

#### 4.0 SUMMARY

1) The autocovariance function of the LANDSAT-D accelerations indicated that the steady state sigmas for the three components were approximately  $24 \times 10^{-6}$  (H),  $7 \times 10^{-6}$  (C) and  $10 \times 10^{-6}$  (L) m/sec<sup>2</sup>. When modeling these accelerations as first order Markov processes, time constants of 1200 (H), 200 (C) and 200 (C) seconds should be used. However, when these values were used in PREFER, better results were obtained when smaller values of the steady state sigmas were used (up to an order of magnitude smaller). It is believed that the first order Markov acceleration model (used in PREFER) can only approximately match the characteristics of the true accelerations and thus there is a discrepancy between the process magnitudes computed by the two methods. However, the PREFER results are not particularly sensitive to the exact values of these parameters if they are reasonable.

2) Station positions errors, measurement biases and refraction parameters should be estimated in PREFER. However, the station positions are only weakly observable and thus the estimated values cannot be relied upon as accurate estimates.

3) The PREFER solution is not sensitive to assumptions of *a priori* error statistics or to the mini-batch step size (within reasonable limits).

4) The nominal trajectory used as input to PREFER should be as accurate as possible.

5) The log likelihood function and sum of weighted residuals can be used as reasonably reliable indicators of the optimum model when comparing runs using different model assumptions. In particular, these metrics are quite sensitive to errors in the measurement models (e.g. noise sigmas, bias sigmas, etc.).

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5.0 REFERENCES

- 1) Gibbs, B. P., "Evaluation of Landsat-D Orbit Determination Using a Filter/Smoother (PREFER)", Business and Technological Systems, Inc., BTS-FR-81-146, March 1981.

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